GRADE 9-10 QUESTIONS AND SOLUTIONS

Q1: Given x, y, z as natural numbers:

$x.y = 8$
$y.z = 4$

What is the smallest possible value of x + y + z ? (1 point)

A) 12 B) 10 C) 8 D) 7 E) 6

Q2: Given:

$x < y < 0 < z$

Which of the following is positive? (1 point)

A) $x^2 . y . z$
B) $x^3 . y^2 . z$
C) $z . y$
D) $x . z^3$
E) $x . y . z$

Q3: The sum of the first four consecutive even numbers out of five is 116. What is the fourth number? (1 point)

A) 24 B) 26 C) 28 D) 30 E) 32

Q4: $x = 3 + 6 + 9 + \ldots + 30$
$y = 5 + 10 + 15 + \ldots + 55$

What is 2x – y? (1 point)

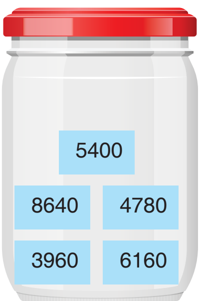
A) 0 B) 10 C) 45 D) 55 E) 66

Q5: $(x - y + 2)^2 + (x + y + 8)^2 = 0$

Find the value of y. (1 point)

A) –5 B) –3 C) –1 D) 0 E) 3

Q6:

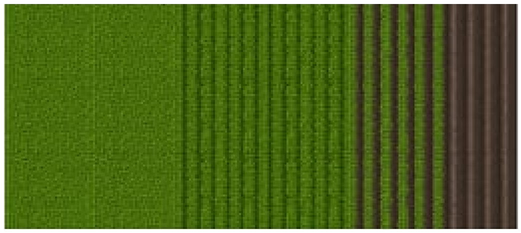


From the jar containing five numbers, one number is selected. It is known that the selected number is divisible by 5, 8, 9, and 11 without a remainder.

Which number is it? (2 points)

A) 5400
B) 8640
C) 4780
D) 3960
E) 6160

Q7:

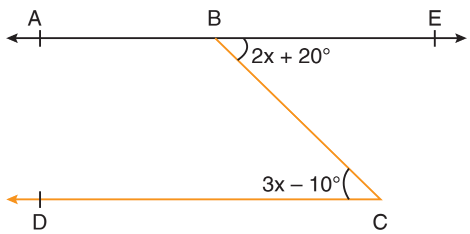


A rectangular field with a width of 65 cm and a length of 90 cm will have trees planted around its perimeter at equal intervals, including at the corners.

What is the minimum number of trees required for this? (2 points)

A) 62 B) 31 C) 18 D) 13 E) 5

Q8:



In the figure, AE // CD.

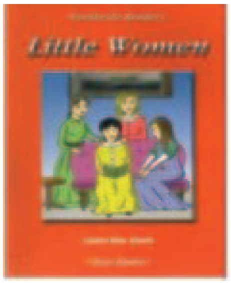
The measures of the angles are given as:

$m(\widehat{EBC}) = 2x + 20°$ and $m(\widehat{BCD}) = 3x - 10°$.

What is the measure of $m(\widehat{ABC})$? (2 points)

A) 90°
B) 95°
C) 100°
D) 110°
E) 120°

Q9:



Matthew reads 3 more pages each day than the previous day.

If he reads 6 pages on the first day, how many days will it take him to finish a 510-page book? (2 points)

A) 17 B) 18 C) 19 D) 20 E) 21

Q10: For $x > 5$, the three-digit number y9x is divisible by 6.

How many different values can y take? (2 points)

A) 7 B) 6 C) 5 D) 4 E) 3

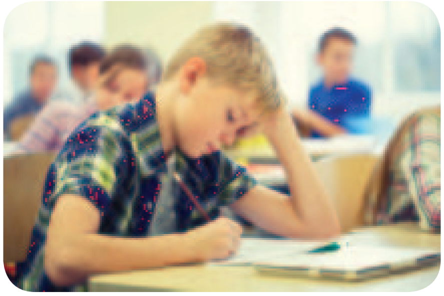
Q11: $2^x.5^y = 25$

$2^y.5^x = 4$

What is the value of x + y? (3 points)

A) 0 B) 1 C) 2 D) 4 E) 5

Q12:

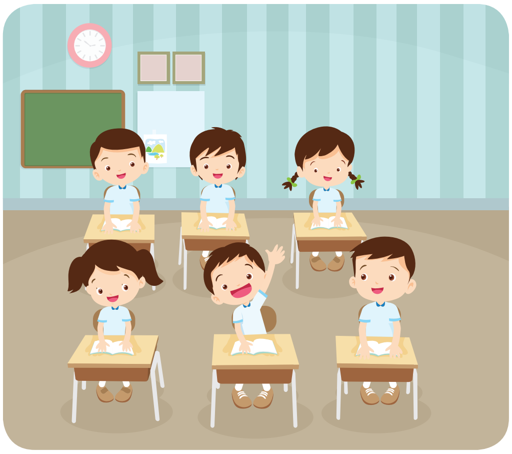


A student solves 150 questions in 135 minutes.

How many minutes will it take the student to solve 100 questions? (3 points)

A) 50 B) 60 C) 75 D) 90 E) 100

Q13:



There are 33 students in a classroom, consisting of boys and girls. If 4 girls leave the class and 3 boys join, the number of boys and girls becomes equal.

How many girls were there in the class initially? (3 points)

A) 13 B) 16 C) 17 D) 18 E) 20

Q14: Bella says that 14 more than 3 times a number is equal to 5 more than 4 times the number decreased by 3.

What is the number Bella is talking about? (3 points)

A) 20 B) 21 C) 22 D) 23 E) 24

Q15:

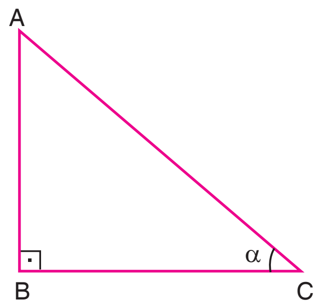
Monday	Tuesday	Wednesday	Thursday	Friday
22°	25°	17°	19°	32°

The table shows the temperatures in Rome over five days.

Which of the following statements is incorrect? (3 points)

A) The range is 15°.
B) The mode is 0°.
C) The median is 22°.
D) The hottest day is Friday.
E) The coldest day is Wednesday.

Q16:



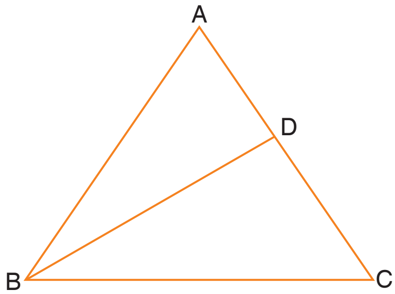
In the right triangle ABC:

- $AB \perp BC$
- $m(\widehat{ACB}) = \alpha$
- $|AB| = 9\,cm$
- $|BC| = 12\,cm$

What is $\sin\alpha - \cos\alpha$? (4 points)

A) $-\frac{3}{4}$ B) $-\frac{2}{5}$ C) $-\frac{1}{5}$ D) $\frac{1}{5}$ E) $\frac{2}{5}$

Q17:



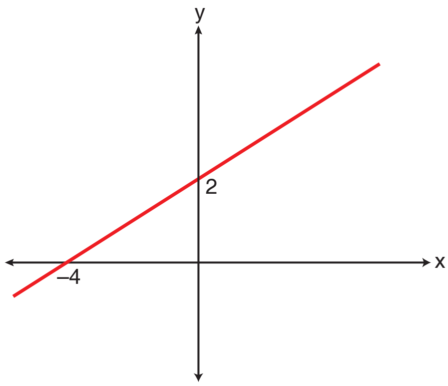
In triangle ABC:

- $BD \perp AC$
- $|AD| = 9\,cm$
- $|AB| = 15\,cm$
- $|BC| = 20\,cm$

What is the area A(ABC)?

A) 120 cm^2 B) 130 cm^2
C) 140 cm^2 D) 150 cm^2
E) 160 cm^2

Q18:

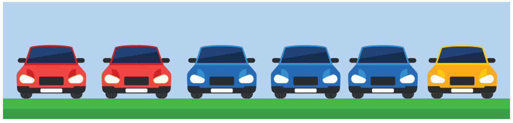


The graph of the function y = f(x) is shown.

What is the value of f(–2) + f(0) + f(3)? (4 points)

A) $\frac{9}{2}$ B) $\frac{11}{2}$ C) $\frac{13}{2}$ D) $\frac{15}{2}$ E) $\frac{17}{2}$

Q19:



What is the probability that the 3 blue cars are next to each other? (4 points)

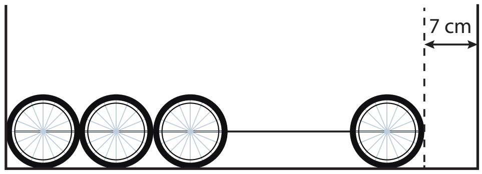
A) $\frac{1}{5}$ B) $\frac{1}{6}$ C) $\frac{1}{8}$ D) $\frac{1}{10}$ E) $\frac{1}{12}$

Q20:

From 4 married couples, how many different 3-person groups can be formed such that one married couple is included in the group? (4 points)

A) 16 B) 20 C) 24 D) 28 E) 32

Q21:



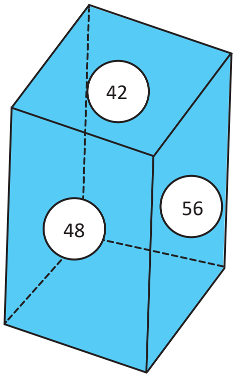
The length of the box is given as

$2x^3 + 10x^2 + 5x + 17$ cm, and it contains identical toy car wheels with a radius of $x^2 + 2$ cm. When the wheels are placed side by side in the box, there is a 7 cm gap left between the last wheel and the box.

What is the value of a, the number of wheels that fit into the box? (5 points)

A) 8 B) 10 C) 12 D) 15 E) 16

Q22:



The figure is a rectangular prism with three different face areas given as $42\,cm^2$, $48\,cm^2$ and $56\,cm^2$.

What is the volume of this rectangular prism? (5 points)

A) $315\ cm^3$ B) $336\ cm^3$
C) $357\ cm^3$ D) $378\ cm^3$
E) $432\ cm^3$

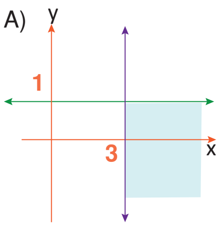
Q23: The numbers 5^1, 5^2, 5^3, 5^4, 5^5 are given.

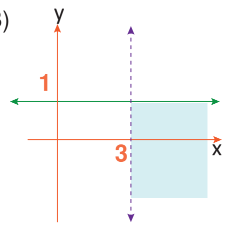
What is the geometric mean of these numbers? (5 points)

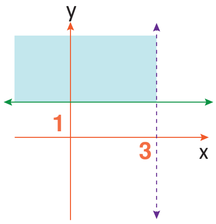
A) 5 B) 25 C) 100 D) 120 E) 125

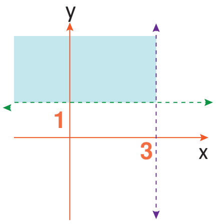
Q24: The inequalities $x > 3$ and $y \leq 1$ describe a region in the coordinate plane.

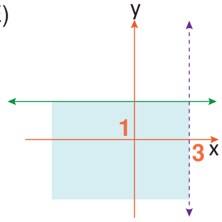
Which of the shaded regions in the options corresponds to this system of inequalities? (5 points)

A) 

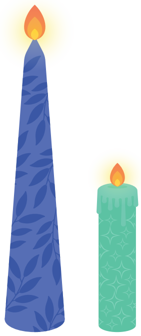
B) 

C) 

D) 

E) 

Q25:



The burning rates of two candles are proportional to 1 and 2, respectively. Their lengths are proportional to 3 and 4, respectively. After being lit at the same time, the lengths of the two candles are equal after 2 hours.

How many hours will it take for the shorter candle to burn out? (5 points)

A) 10 B) 9 C) 8 D) 6 E) 5

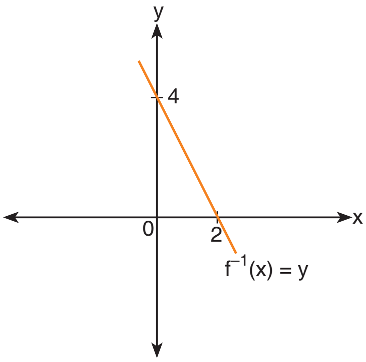
Q26:

x kg of salt and y kg of flour are mixed together.

What percentage of the mixture is salt? (6 points)

A) $\frac{x}{x+y}$ B) $\frac{y}{x+y}$ C) $\frac{100.y}{x+y}$ D) $\frac{100.x}{x+y}$ E) $\frac{x}{100(x+y)}$

Q27:



The graph of the inverse function $f^{-1}(x)$ is shown.

What is the value of a that satisfies $f(a) = -4$? (6 points)

A) 8 B) 10 C) 12 D) 14 E) 16

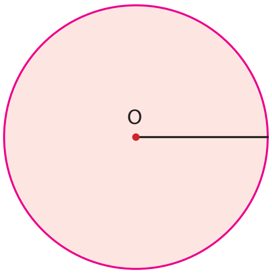
Q28: For the quadratic equation $a^2 + bx + 6 = 0$, let x_1 and x_2 be its roots, and the following conditions hold:

1. $a,b \in R$
2. $x_1 + x_2 = -3$
3. $x_1 . x_2 = -2$

What is the value of a – b? (6 points)

A) –12 B) –6 C) 0 D) 6 E) 12

Q29:



The area of the circle centered at O is given as $3x^4 - 12x^2 + 12$ square meters.

What is the remainder when the radius of this circle is divided by (x – 1), given that π = 3? (6 points)

A) –3 B) –2 C) –1 D) 0 E) 1

Q30: The sequence [a, b] represents a sequence of b consecutive integers where a is the median.

For the sequence formed by combining the sequences [2, 5] and [4, 7], evaluate the following statements:

I. The sequence has no mode.

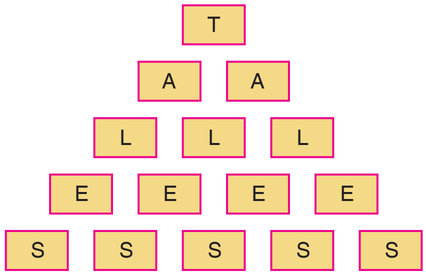
II. The median of the sequence is 3.

III. The arithmetic mean of the sequence is 3.

Which of the above statements are correct? (6 points)

A) Only I B) Only II C) I and II D) II and III E) I, II and III

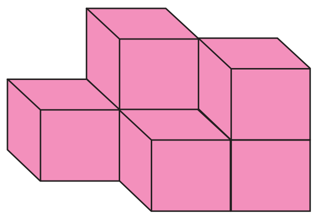
Q31:



By following the adjacent letters from top to bottom, how many different ways can the word "TALES" be read? (7 points)

A) 8 B) 10 C) 12 D) 14 E) 16

Q32:



In the given figure, there is a structure made up of 6 unit cubes.

Accordingly, what is the surface area of this structure in square units? (7 points)

A) 26 B) 28 C) 30 D) 32 E) 34

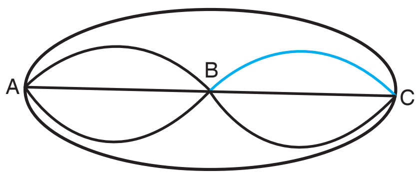
Q33: Given the function:

$f: [2, \infty) \rightarrow [-3, \infty)$
$f(x) = x^2 - 4x + 1$

Which of the following is the inverse function $f^{-1}(x)$? (7 points)

A) $\sqrt{x+3}$
B) $\sqrt{x-3}$
C) $\sqrt{x-3}+2$
D) $\sqrt{x-3}-2$
E) $\sqrt{x+3}+2$

Q34:

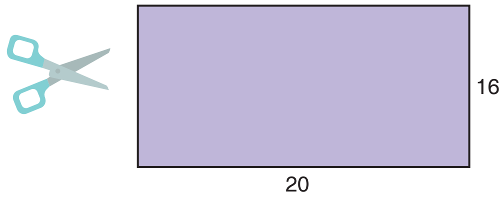


The figure shows the routes between cities A, B, and C. It is known that a vehicle traveling from city A to city C passes through city B.

What is the probability that the vehicle takes the blue road? (7 points)

A) $\frac{1}{3}$
B) $\frac{1}{5}$
C) $\frac{2}{5}$
D) $\frac{3}{7}$
E) $\frac{1}{9}$

Q35:



A rectangular cardboard with dimensions 16 cm and 20 cm is folded to form a closed square prism of height 6 cm. No part of the cardboard is wasted during this process.

What is the volume of the square prism in cubic centimeters? (7 points)

A) 384 cm^3
B) 408 cm^3
C) 486 cm^3
D) 600 cm^3
E) 726 cm^3

GRADE 9-10 QUESTIONS AND SOLUTIONS

ANSWER IS D

SOLUTION:

Q1: To minimize x + y + z, let's analyze the equations.

Step 1: Factor x.y = 8 in natural numbers

The possible pairs (x, y) are:

- (1, 8)
- (2, 4)
- (4, 2)
- (8, 1)

Step 2: Factor y.z = 4 in natural numbers

For each y from Step 1:

- If y = 8, z.8 = 4: No solutions (not a natural number).
- If y = 4, z.4 = 4: z = 1.
- If y = 2, z.2 = 4: z = 2.
- If y = 1, z.1 = 4.

Step 3: Minimize x + y + z

Now calculate x + y + z for valid combinations:

- For (x, y, z) = (2, 4, 1): x + y + z = 2 + 4 + 1 = 7.
- For (x, y, z) = (4, 2, 2): x + y + z = 4 + 2 + 2 = 8.
- For other cases, x + y + z is larger.

Conclusion:

The smallest value of x + y + z is 7.

ANSWER IS E

SOLUTION:

Q2: x and y are **negative**, with x < y. z is **positive**.

Option A: $x^2.y.z$

- x^2 is positive (square of a negative number).
- y is negative.
- z is positive.
- $x^2.y.z$ = positive.negative.positive = negative

Option B: $x^3.y^2.z$

- x^3 is negative (cube of a negative number).
- y^2 is positive (square of a negative number).
- z is positive.
- $x^3.y^2.z$ = negative.positive.positive = negative

Option C: y.z

- z is positive.
- y is negative.
- y.z = positive.negative = negative

Option D: $x.^3$

- x is negative.
- z^3 is positive (cube of a positive number).
- $x.z^3$ = negative.positive = negative

Option E: x.y.z

- x is negative.
- y is negative.
- z is positive.
- x.y.z = negative.negative.positive = positive

The only positive expression is E: x.y.z.

ANSWER IS E

SOLUTION:

Q3: Let the consecutive even numbers be:

$x, x + 2, x + 4, x + 6, x + 8$

The sum of the first four numbers is given as:

$x + (x + 2) + (x + 4) + (x + 6) = 116$

$4x + 12 = 116$

$4x = 104$

$x = 26$

The fourth number is: $x + 6 = 26 + 6 = 32$

ANSWER IS A

SOLUTION:

Q4: For $x = 3 + 6 + 9 + \ldots + 30$

This is an arithmetic sequence with:

- First term (a) = 3
- Common difference (d) = 3
- Last term = 30

For $y = 5 + 10 + 15 + \ldots + 55$

This is an arithmetic sequence with:

- First term (a) = 5
- Common difference (d) = 5
- Last term = 55

For x:

$a_n = a + (n - 1)d$

$30 = 3 + (n - 1)3$

$30 = 3n$

$n = 10$

For y:

$a_n = a + (n - 1)d$

$55 = 5 + (n - 1)5$

$55 = 5n$

$n = 11$

The sum of an arithmetic sequence is:

$S_n = \frac{n}{2}(a + l)$

where n is the number of terms, a is the first term, and l is the last term.

For x:

$S_x = \frac{10}{2}(3 + 30) = 5.33 = 165$

For y:

$S_y = \frac{11}{2}(5 + 55) = \frac{11}{2}.60 = 11.30 = 330$

Calculate $2x - y$

$2x - y = 2(165) - 330 = 330 - 330 = 0$

ANSWER IS B

SOLUTION:

Q5: For the sum of two squares to equal zero, each term must independently equal zero. Therefore, we set:

$x - y + 2 = 0 \Rightarrow x - y = -2 \Rightarrow x = y - 2$

$x + y + 8 = 0 \Rightarrow x + y = -8$

Substitute $x = y - 2$ into $x + y = -8$

$(y - 2) + y = -8 \Rightarrow 2y - 2 = -8 \Rightarrow 2y = -6$

$\Rightarrow y = -3$

Therefore, the answer is B.

ANSWER IS D

SOLUTION:

Q6: We need to find the number in the list that is divisible by 5, 8, 9, and 11. This means the number must be divisible by the LCM of these numbers.

To find the LCM:

- $5 = 5$
- $8 = 2^3$
- $9 = 3^2$
- $11 = 11$

$LCM = 2^3.3^2.5.11 = 3960$

So, the LCM of 5, 8, 9, and 11 is 3960.

We divide each number by 3960 to see if it is divisible (no remainder):

$5400 \div 3960 \neq$ integer

$8640 \div 3960 \neq$ integer

$4780 \div 3960 \neq$ integer

$3960 \div 3960 = 1$ (integer)

$6160 \div 3960 \neq$ integer

The only number that works is 3960. Therefore, the correct answer is D.

ANSWER IS A

SOLUTION:

Q7: The perimeter of a rectangle is given by:

Perimeter = 2.(length + width)

In here:

Perimeter = 2.(90 + 65) = 2.155 = 310 cm

To minimize the number of trees, the distance between trees (interval) should be the greatest possible value that divides the total perimeter (310 cm) evenly. This is equivalent to finding the greatest common factor (GCF) of the length (90 cm) and width (65 cm).

- Prime factorization of 90: $90 = 2.3^2.5$
- Prime factorization of 65: $65 = 5.13$
- Common factor: 5

Thus, the GCF is 5 cm.

If the interval is 5 cm, the number of intervals around the perimeter is:

$$\text{Number of intervals} = \frac{\text{Perimeter}}{\text{Interval}} = \frac{310}{5} = 62$$

Since the trees are planted at every interval, including the starting point, the number of trees required is 62.

ANSWER IS C

SOLUTION:

Q8: Since the two lines are parallel, we can equate the angles:

$2x + 20° = 3x - 10°$

Subtract 2x from both sides:

$2x - 2x + 20° = 3x - 2x - 10°$
$20° = x - 10°$

Add 10° to both sides:

$20° + 10° = x - 10° + 10°$
$x = 30°$

Now, calculate the value of 2x + 20°:

$2(30°) + 20° = 60° + 20° = 80°$

Since ABE is a straight line, the sum of $\widehat{ABC}$ and $\widehat{EBC}$ is 180°.
Given that $\widehat{EBC} = 80°$, we find:
$\widehat{ABC} = 180° - 80° = 100°$.

ANSWER IS A

SOLUTION:

Q9: We are solving how many days Matthew will take to finish a 510 – page book, given:

He reads 6 pages on the first day.

Each day, he reads 3 more pages than the previous day.

This problem involves an arithmetic sequence where:

- The first term (a = 6) represents the pages Matthew reads on the first day.
- The common difference (d = 3) represents how much more he reads each day.
- The total number of pages read is 510 ($S_n = 510$).

We need to find the number of days (n).

The sum of the first n terms of an arithmetic sequence is:

$$S_n = \frac{n}{2} \times \big(2a + (n - 1)d\big)$$

Substitute $S_n = 510$, a = 6, and d = 3:

$$510 = \frac{n}{2} \times \big(2(6) + (n - 1)(3)\big)$$

Simplify:

$$510 = \frac{n}{2} \times (12 + 3n - 3)$$

$$510 = \frac{n}{2} \times (3n + 9)$$

Multiply both sides by 2 to get rid of the fraction:

$1020 = n(3n + 9)$

Expand: $1020 = 3n^2 + 9n$

Rearrange the equation:

$3n^2 + 9n - 1020 = 0$

Simplify by dividing through by 3:

$n^2 + 3n - 340 = 0$

We need to find n (a whole number) such that:

$n^2 + 3n - 340 = 0$

Check n = 17:

$17^2 + 3(17) - 340 = 289 + 51 - 340 = 0.$

So, n = 17 is correct.

ANSWER IS B

SOLUTION:

Q10: Divisibility rule for 6:

For a number to be divisible by 6, it must satisfy two conditions:

1. The number must be divisible by 2 (even).
2. The number must be divisible by 3 (the sum of its digits must be divisible by 3).

Divisible by 2:

The last digit x must be even. Since $x > 5$, the possible values for x are:

$x = 6$ or 8.

Divisible by 3:

The sum of the digits $y + 9 + x$ must be divisible by 3.

Case 1: x = 6

- The sum of the digits is: $y + 9 + 6 = y + 15$.
- For $y + 15$ to be divisible by 3: $y + 15.0(\text{mod}3)$

Possible values for y (as a nonzero digit) are: $y = 3, 6, 9$. So, there are 3 possible values for y when $x = 6$.

Case 2: x = 8

- The sum of the digits is: $y + 9 + 8 = y + 17$.
- For $y + 17$ to be divisible by 3: $y + 17.0(\text{mod}3)$.

Possible values for y (as a nonzero digit) are: $y = 1,4,7$. So, there are 3 possible values for y when $x = 8$.

From both cases:

- 3 values for y when $x = 6$,
- 3 values for y when $x = 8$.

The total number of different values y can take is 6.

ANSWER IS C

SOLUTION:

Q11: Write the equations in terms of powers of 2 and 5:

From the first equation:

$2^x.5^y = 25$

Since $25 = 5^2$, this becomes:

$2^x.5^y = 5^2$

Therefore:

$2^x = 5^{2-y}$

From the second equation:

$2^y 5^x = 4$

Since $4 = 2^2$, this becomes:

$2^y.5^x = 2^2$

Therefore:

$5^x = 2^{2-y}$

Take the logarithmic equality between the powers of 2 and 5.

The solution to the system of equations is:

$x = 0$ and $y = 2$

$x + y = 0 + 2 = 2$

ANSWER IS D

SOLUTION:

Q12: The student solves:

$$\text{Questions per minute} = \frac{\text{Total questions}}{\text{Total time}} = \frac{150}{135}$$

Simplify:

$$\frac{150}{135} = \frac{10}{9}$$

So, the student solves $\frac{10}{9}$ questions per minute.

Let the time to solve 100 questions be x minutes.

Using the rate of $\frac{10}{9}$ questions per minute:

$$\frac{10}{9} \times x = 100.$$

Solve for x:

$$x = 100 \times \frac{9}{10} = 90$$

It will take the student 90 minutes to solve 100 questions.

ANSWER IS E

SOLUTION:

Q13: Let:

- g = number of girls initially,
- b = number of boys initially.

We know:

Total students: g + b = 33.

After 4 girls leave and 3 boys join, the number of boys and girls becomes equal: g − 4 = b + 3

From the second equation:

$$g - 4 = b + 3$$

Simplify:

$$g = b + 7$$

Substitute g = b + 7 into g + b = 33:

$$(b + 7) + b = 33.$$

Simplify:

$$2b + 7 = 33$$
$$2b = 26$$
$$b = 13$$

Substitute b = 13 into g = b + 7:

$$g = 13 + 7 = 20$$

Initially: g = 20, b = 13, total = 20 + 13 = 33.

The initial number of girls is 20.

ANSWER IS B

SOLUTION:

Q14: Let the number be x. Based on the problem:

$$3x + 14 = 4(x - 3) + 5$$

Expand both sides:

$$3x + 14 = 4x - 12 + 5$$
$$3x + 14 = 4x - 7$$

Rearrange to isolate x:

$$3x - 4x = -7 - 14$$
$$-x = -21$$

Solve for x: $x = 21$

The answer is B.

ANSWER IS B

SOLUTION:

Q15: Calculate the range (A):

Range = Maximum temperature − Minimum temperature

$$\text{Range} = 32° - 17° = 15°$$

Statement A is correct.

Determine the mode (B): The mode is the value that appears most frequently.

Temperatures: 22, 25, 17, 19, 32 (all unique).

No value repeats, so there is no mode.

Statement B is incorrect because it states the mode is 0°.

Find the median (C): Arrange the temperatures in ascending order:

17, 19, 22, 25, 32

The median (middle value) is 22.

Statement C is correct.

Identify the hottest day (D): The hottest temperature is 32° on Friday.

Statement D is correct.

Identify the coldest day (E): The coldest temperature is 17° on Wednesday.

Statement E is correct.

ANSWER IS C

SOLUTION:

Q16: Find the hypotenuse AC: Use the Pythagorean theorem:

$$AC = \sqrt{AB^2 + BC^2} = \sqrt{9^2 + 12^2} = \sqrt{81 + 144}$$
$$= \sqrt{225} = 15$$

Calculate sinα and cosα:

- $\sin\alpha = \frac{\text{Opposite side}}{\text{Hypotenuse}} = \frac{AB}{AC} = \frac{9}{15} = \frac{3}{5}$
- $\cos\alpha = \frac{\text{Adjacent side}}{\text{Hypotenuse}} = \frac{BC}{AC} = \frac{12}{15} = \frac{4}{5}$

Compute sinα – cosα:

$$\sin\alpha - \cos\alpha = \frac{3}{5} - \frac{4}{5} = \frac{-1}{5}$$

The answer is C.

ANSWER IS D

SOLUTION:

Q17: To find the area of $\widehat{ABC}$, we use the formula for the area of a triangle:

$$A(ABC) = \frac{1}{2}.\text{base}.\text{height}.$$

Base and Height:

- The base is AC, which needs to be calculated.
- The height is BD, which we will determine using Pythagoras.

Since AB = 15 and BD is perpendicular to AC, we focus on $\widehat{ABD}$. Using the Pythagorean theorem:

$$AB^2 = AD^2 + BD^2$$

Substitute the known values:

$$15^2 = 9^2 + BD^2$$
$$225 = 81 + BD^2$$
$$BD^2 = 225 - 81 = 144$$
$$BD = \sqrt{144} = 12$$

Since AC = AD + DC, and DC can be determined using the Pythagorean theorem in $\widehat{BDC}$, calculate DC:

$$BC^2 = BD^2 + DC^2$$
$$20^2 = 12^2 + DC^2$$
$$400 = 144 + DC^2$$
$$DC = \sqrt{400 - 144} = \sqrt{256} = 16$$
$$AC = AD + DC = 9 + 16 = 25.$$

Calculate the area A(ABC):

$$A(ABC) = \frac{1}{2}.AC.BD = \frac{1}{2}.25.12 = 150\,cm^2.$$

ANSWER IS C

SOLUTION:

Q18: From the graph:

- The line passes through (−4, 0) and (0, 2), so we calculate the slope (m):

$$m = \frac{\Delta y}{\Delta x} = \frac{2-0}{0-(-4)} = \frac{2}{4} = \frac{1}{2}$$

- Using the slope-intercept form $y = mx + b$, where b is the y–intercept:

$$f(x) = \frac{1}{2}x + 2$$

Calculate f(−2):

$$f(-2) = \frac{1}{2}(-2) + 2 = -1 + 2 = 1$$

Calculate f(0):

$$f(0) = \frac{1}{2}(0) + 2 = 2$$

Calculate f(3):

$$f(3) = \frac{1}{2}(3) + 2 = \frac{3}{2} + 2 = \frac{3}{2} + \frac{4}{2} = \frac{7}{2}$$

Sum the values:

$$f(-2) + f(0) + f(3) = 1 + 2 + \frac{7}{2} = \frac{2}{2} + \frac{4}{2} + \frac{7}{2} = \frac{13}{2}$$

The answer is C.

ANSWER IS A

SOLUTION:

Q19: We are arranging 6 cars:

- 2 red cars (identical),
- 3 blue cars (identical),
- 1 yellow car.

We want to calculate the probability that the 3 blue cars are adjacent.

The total number of ways to arrange 6 cars, accounting for identical cars, is:

$$\text{Total Arrangements} = \frac{6!}{2!.3!} = \frac{720}{12} = 60$$

If the 3 blue cars are next to each other, treat them as a single block. Now, we are arranging:

- 1 blue block,
- 2 red cars,
- 1 yellow car.

This is equivalent to arranging 4 items:

$$\text{Arrangements of 4 items} = \frac{4!}{2!} = \frac{24}{2} = 12$$

Since the 3 blue cars are identical, there is only 1 way to arrange them within the block.

Thus, the total number of favorable arrangements (where blue cars are adjacent) is 12.

The probability of the 3 blue cars being adjacent is:

$$\text{Probability} = \frac{\text{Favorable Arrangements}}{\text{Total Arrangements}} = \frac{12}{60} = \frac{1}{5}.$$

ANSWER IS C

SOLUTION:

Q20: We need to create 3-person groups where **1 married couple** is always included, and the third person is chosen from the remaining individuals.

There are 4 married couples to choose from. The number of ways to choose **1 married couple** is:

$$\binom{4}{1} = 4$$

After selecting a married couple, there are 6 remaining individuals (8 total people minus the 2 in the selected couple). The third person must be chosen from these 6 individuals:

$$\binom{6}{1} = 6$$

The total number of ways to form a group is the product of the two choices:

$$4.6 = 24$$

The answer is C.

ANSWER IS D

SOLUTION:

Q21: We are given:

- The **length of the box:** $2x^3 + 10x^2 + 5x + 17$ cm,
- The **radius** of each wheel: $x^2 + 2$ cm,
- The **diameter** of each wheel (since wheels are placed side by side): Diameter = 2.Radius = $2(x^2 + 2) = 2x^2 + 4$ cm
- There is a **7 cm gap** left after a wheels are placed.

The total length occupied by a wheels plus the gap equals the total box length:

$$a.(2x^2 + 4) + 7 = 2x^3 + 10x^2 + 5x + 17$$

Simplify:

$$a.(2x^2 + 4) = 2x^3 + 10x^2 + 5x + 10$$

Factorize $2x^2 + 4$ out of the right-hand side to express a:

$$a = \frac{x^3 + 10x^2 + 5x + 10}{2x^2 + 4}$$

Divide $2x^3 + 10x^2 + 5x + 10$ by $2x^2 + 4$:

First term:

Divide $2x^3$ by $2x^2$: x

Multiply x by $2x^2 + 4$: $2x^3 + 4x$

Subtract:

$$(2x^3 + 10x^2 + 5x + 10) - (2x^3 + 4x) = 10x^2 + x + 10$$

Second term:

Divide $10x^2$ by $2x^2$: 5

Multiply 5 by $2x^2 + 4$: $10x^2 + 20$

Subtract: $(10x^2 + x + 10) - (10x^2 + 20) = x - 10$

The result is: $a = x + 5 + \frac{x - 10}{2x^2 + 4}$

For a to be an integer, the remainder $\frac{x - 10}{2x^2 + 4}$ must equal 0. This happens when:

$$x - 10 = 0 \Rightarrow x = 10$$

If x = 10, then:

$$a = x + 5 = 10 + 5 = 15$$

The answer is D.

ANSWER IS B

SOLUTION:

Q22: We are given the face areas:

- $ab = 42$
- $bc = 48$
- $ca = 56$

Now, factorize each of these:

- $42 = 6 \times 7$
- $48 = 6 \times 8$
- $56 = 7 \times 8$

So, we can assign:

$a = 6, b = 7, c = 8$

The volume of the prism is:

Volume = $a.b.c = 6.7.8.$

First, multiply:

$6.\ 7 = 42$

Then:

$42.8 = 336$

The volume of the prism is 336 cm^3.

ANSWER IS E

SOLUTION:

Q23: The geometric mean of n numbers $a_1, a_2, \ldots, a_n$ is calculated as:

Geometric Mean = $\sqrt[n]{a_1.a_2 \ldots . a_n}$

For the given numbers $5^1, 5^2, 5^3, 5^4, 5^5$:

Geometric Mean = $\sqrt[5]{5^1.5^2.5^3.5^4.5^5}$

Using the property of exponents:

$5^1.5^2.5^3.5^4.5^5 = 5^{1+2+3+4+5}$

Calculate the sum of exponents:

$1 + 2 + 3 + 4 + 5 = 15$

Thus:

$5^{1+2+3+4+5} = 5^{15}$

The geometric mean is:

Geometric Mean = $\sqrt[5]{5^{15}} = 5^{\frac{15}{5}} = 5^3$

$5^3 = 5.5.5 = 125$

The answer is E.

ANSWER IS B

SOLUTION:

Q24: This inequality represents all points to the right of the vertical line $x = 3$, excluding the line itself. This means:

- x values are greater than 3,
- The vertical boundary line $x = 3$ is not shaded (it is dotted).

This inequality represents all points below or on the horizontal line $y=1$. This means:

- y values are less than or equal to 1,
- The horizontal boundary line $y = 1$ is included (it is solid).

The region satisfying both inequalities is:

- To the right of $x = 3$ (but not including the line $x = 3$),
- Below or on $y = 1$

This forms a rectangular region bounded on the left by a dotted line $x = 3$ and on the top by a solid line $y = 1$. The shading occurs in the bottom-right portion of the plane.

The correct answer is Option B.

ANSWER IS D

SOLUTION:

Q25: Let the burning rates of the candles be:

- x for the first candle (burning rate proportional to 1),
- $2x$ for the second candle (burning rate proportional to 2).

Let the initial lengths of the candles be:

- $3k$ for the first candle,
- $4k$ for the second candle.

The lengths of the candles decrease at their respective burning rates. After 2 hours:

- The length of the first candle: Length $= 3k - 2x$
- The length of the second candle: Length $= 4k - 2(2x) = 4k - 4x$

The problem states that the lengths of the two candles are equal after 2 hours:

$$3k - 2x = 4k - 4x$$

Rearrange:

$$k = 2x$$

The shorter candle initially has a length of $3k$, and its burning rate is x. The time it takes to burn out is:

$$t = \frac{\text{Initial Length}}{\text{Burning Rate}} = \frac{3k}{k}$$

Substitute $k = 2x$:

$$t = \frac{3(2x)}{x} = 6$$

The shorter candle burns out in 6 hours.

ANSWER IS D

SOLUTION:

Q26: The total weight of the mixture is:

Total Weight = x + y

The weight of salt is x. The proportion of salt in the mixture is:

$$\text{Proportion of Salt} = \frac{x}{x + y}$$

To express this proportion as a percentage:

$$\text{Percentage of Salt} = \frac{x}{x + y}.100$$

The percentage of salt in the mixture is:

D) $\frac{100.x}{x + y}$

ANSWER IS C

SOLUTION:

Q27: The graph given is for $f^{-1}(x)$.

For any f(x) and its inverse $f^{-1}(x)$:

$$f\left(f^{-1}(x)\right) = x \text{ and } f^{-1}\left(f(x)\right) = x.$$

This means that the roles of x and y are swapped in f(x) and $f^{-1}(x)$.

If f(a) = −4, then from the inverse function:

$$f^{-1}(4) = a$$

Our task is to determine the x-coordinate on the graph of $f^{-1}(x)$ where y = −4.

From the graph:

The line passes through points (2,0) and (0,4).

The slope m of the line is:

$$m = \frac{\Delta y}{\Delta x} = \frac{4 - 0}{0 - 2} = -2$$

Thus, the equation of the line f^(-1)(x) is:

$$f^{-1}(x) = -2x + 4$$

Using the relationship $f^{-1}(4) = a$, substitute y = −4 into $f^{-1}(x)$:

$$-4 = -2a + 4$$

Simplify and solve for a:

$$-2a = -4 - 4$$
$$-2a = -8$$
$$a = 4$$

The graph shows that when $f^{-1}(x) = -4$,

the corresponding x-coordinate is y = 12.

ANSWER IS D

SOLUTION:

Q28: Step 1: Use Vieta's Formulas

For a quadratic equation $ax^2 + bx + c = 0$, Vieta's formulas state:

$$x_1 + x_2 = \frac{-b}{a}$$

$$x_1.x_2 = \frac{c}{a}$$

Here, c = 6. Using the given conditions:

$$x_1 + x_2 = -3$$

$$\frac{-b}{a} = -3 \Rightarrow \frac{b}{a} = 3$$

$$x_1.x_2 = -2$$

$$\frac{c}{a} = -2 \Rightarrow \frac{6}{a} = -2$$

Step 2: Solve for a and b

From $\frac{6}{a} = -2$

$$a = \frac{6}{-2} = -3$$

From $\frac{b}{a} = 3$

$$b = 3a = 3(-3) = -9$$

Step 3: Calculate a–b

Substitute a = –3 and b = –9:

$$a - b = -3 - (-9) = -3 + 9 = 6$$

ANSWER IS C

SOLUTION:

Q29: The area of a circle is given by:

Area = πr^2, where r is the radius.

Using $\pi = 3$, the equation becomes Area = $3r^2$.

From the problem, the area is also given as $3x^4 - 12x^2 + 12$. Equating the two expressions for the area:

$$3r^2 = 3x^4 - 12x^2 + 12$$

Step 2: Simplify for r^2

Divide both sides by 3:

$$r^2 = x^4 - 4x^2 + 4$$

Step 3: Factorize r^2

Factorize the expression for r^2:

$$r^2 = (x^2 - 2)^2$$

Thus:

$$r = |x^2 - 2|$$

Step 4: Find the Remainder When r is Divided by (x – 1)

The radius $r = |x^2 - 2|$ simplifies to $r = |x^2 - 2|$ (since radius is positive).

To find the remainder when r is divided by (x – 1), perform polynomial division of $x^2 - 2$ by x – 1.

Step 5: Polynomial Division

Perform the division of $x^2 - 2$ by x – 1.

Steps:

1. Divide x^2 by x to get x,
2. Subtract $x(x - 1) = x^2 - x$.
3. The remainder is –2 + x.

Step 6: Substitute x=1 to Find the Remainder

The remainder is x – 2.

When x = 1:

$$1 - 2 = -1$$

The answer is C.

ANSWER IS B

SOLUTION:

Q30: For {2, 5}:

Since 2 is the median of a sequence of 5 consecutive integers:

Sequence = {0, 1, 2, 3, 4}

For {4, 7}:

Since 4 is the median of a sequence of 7 consecutive integers:

Sequence = {1, 2, 3, 4, 5, 6, 7}

Combine the Sequences

Combine and sort the two sequences:

{0, 1, 1, 2, 2, 3, 3, 4, 4, 5, 6, 7}

Analyze Each Statement

Statement I: "There is no mode."

The mode is the value that appears most frequently.

In the combined sequence {0, 1, 1, 2, 2, 3, 3, 4, 4, 5, 6, 7}, the values 1, 2,3, and 4 each appear twice.

Thus, the sequence has multiple modes.

Statement I is incorrect.

Statement II: "The median is 3."

The median is the middle value when the sequence is sorted.

The combined sequence has 12 elements. The median is the average of the 6th and 7th elements:

$$\text{Median} = \frac{3+3}{2} = 3$$

Statement II is correct.

Statement III: "The arithmetic mean is 3."

The arithmetic mean is the sum of all values divided by the total number of values.

Sum of the sequence:

$0 + 1 + 1 + 2 + 2 + 3 + 3 + 4 + 4 + 5 + 6 + 7 = 38$

Total number of elements: 12

The arithmetic mean is:

$$\text{Mean} = \frac{38}{12} = 3.1667$$

Thus, the arithmetic mean is not exactly 3.

Statement III is incorrect.

ANSWER IS E

SOLUTION:

Q31: At each step:

- From the "T", there are 2 paths to the "A"s.
- From each "A", there are 2 paths to the "L"s (total $2 \times 2 = 4$ paths to the "L"s).
- From each "L", there are 2 paths to the "E"s (total $4 \times 2 = 8$ paths to the "E"s).
- From each "E", there are 2 paths to the "S"s (total $8 \times 2 = 16$ paths to the "S"s).

2.2.2.2=16 ways

Thus, there are 16 distinct ways to read "TALES".

ANSWER IS A

SOLUTION:

Q32: The structure consists of 6 cubes:

- 2 cubes on the top layer.
- 4 cubes on the bottom layer (arranged in a 2×2 square).

We will calculate the visible faces of each cube and count them explicitly.

Top Layer (2 cubes):

Each cube in the top layer has:

- 4 visible side faces (as they are fully exposed on the sides).
- 1 visible top face.
- 1 shared bottom face with the cubes beneath them (this face is hidden).

For 2 cubes in the top layer:

- Visible faces per cube = 4 + 1 =5
- Total visible faces for the top layer = $2 \times 5 = 10$

Bottom Layer (4 cubes):

Each cube in the bottom layer has:

- 3 visible side faces (as they are part of the 2×2 arrangement, sharing faces with adjacent cubes).
- 1 visible bottom face (the structure is not touching the ground).
- 1 hidden face on top (covered by the cubes from the top layer).
- 1 or 2 shared side faces depending on adjacency.

For 4 cubes in the bottom layer:

- Each cube has 3 visible side faces + 1 bottom face = 4 visible faces per cube.
- Total visible faces for the bottom layer = $4 \times 4 = 16$

Adding up the visible faces from both layers:

10 (top layer) +16 (bottom layer) = 26 visible faces.

The visible surface area of the structure is 26 square units.

ANSWER IS E

SOLUTION:

Q33: Step 1: Rewrite y = f(x)

Let y = f(x), so: $y = x^2 - 4x + 1$

Rearrange the equation:

$x^2 - 4x + (1 - y) = 0$

Step 2: Solve for x in terms of y

This is a quadratic equation in x. Using the quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

where a = 1, b = –4, and c = 1 – y. Substitute these values:

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)}}{(1 - y)}$$

Simplify:

$$x = \frac{4 \pm \sqrt{16 - 4(1 - y)}}{2}$$

$$x = \frac{4 \pm \sqrt{16 - 4 \ + 4y)}}{2}$$

$$x = \frac{4 \pm \sqrt{12 + 4y)}}{2}$$

$x = 2 \pm \sqrt{3 + y}$

Step 3: Choose the Correct Branch

The domain of f(x) is $[2, \infty)$, which means $x \geq 2$. For the inverse $f^{-1}(x)$, we need to ensure that $f^{-1}(x)$ is consistent with the increasing nature of the function Thus, we select the positive branch:

$x = 2 + \sqrt{(3 + y)}$

Step 4: Replace y with x

Substitute y with x to express the inverse function:

$f^{-1}(x) = 2 + \sqrt{3 + y}$

ANSWER IS A

SOLUTION:

Q34: Step 1: Total Number of Paths

Each path from A to B can connect to any path from B to C, and based on the figure:

- 3 roads from A to B.
- 3 roads from B to C.

Thus, the total number of paths from A to C is:

Total Paths

= (Paths from A to B) × (Paths from B to C) = 3 × 3 = 9

Step 2: Paths Using the Blue Road

The blue road is 1 of the 3 roads from B to C. For the vehicle to use the blue road:

- It can take any of the 3 roads from A to B.
- It must specifically take the blue road from B to C.

Thus, the number of paths involving the blue road is:

Paths Using Blue Road = 3 × 1 = 3

Step 3: Probability of Taking the Blue Road

The probability is the ratio of paths using the blue road to the total number of paths:

Probability=(Paths Using Blue Road)/(Total Paths)

Substitute the values:

$$\text{Probability} = \frac{3}{9} = \frac{1}{3}$$

ANSWER IS A

SOLUTION:

Q35: The total area of the cardboard is:

Area = 16.20 = 320 cm^2

The surface area of a closed square prism includes:

- **2 square bases (top and bottom):** Each has an area of s^2, where s is the side length of the square.
- **4 vertical rectangular faces:** Each has an area of s.h = s.6

The total surface area is:

Surface Area = $2s^2 + 4(s.6) = 2s^2 + 24s$

Since no part of the cardboard is wasted, the total surface area of the prism equals the area of the cardboard:

$$2s^2 + 24s = 320$$

Rearrange the equation:

$$2s^2 + 24s - 320 = 0$$

Simplify:

$$s^2 + 12s - 160 = 0$$

Solve this quadratic equation.

The solutions to the quadratic equation are:

s = –20 (not valid, as side length cannot be negative) and s = 8. Thus, the side length of the square base is:

$$s = 8\,cm$$

The volume of the square prism is given by:

Volume = Base Area.Height = s^2.h.

Substitute s = 8 and h = 6:

Volume = $8^2.6 = 64.6 = 384\ cm^3$