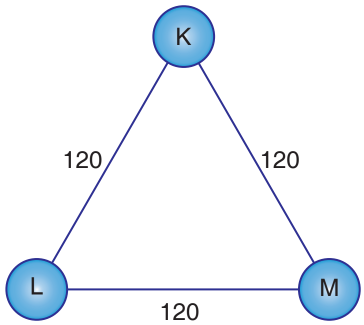
# GRADE 11-12 QUESTIONS AND SOLUTIONS

**Q1:**

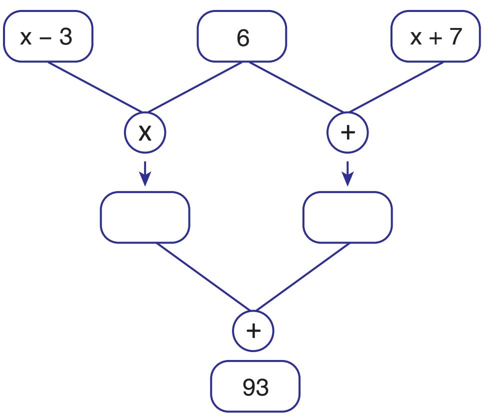



In the given diagram, the numbers in the connected circles represent natural numbers, and the numbers on the edges indicate the Least Common Multiple (LCM) of the natural numbers in the connected circles.

**Given that K, L and M are distinct natural numbers, what is the maximum value of K + L + M?** **(1 point)**

A)360 B) 240 C) 220 D) 210 E) 180

**Q2:**




**What is the value of x?** **(1 point)**

A) 10 B) 11 C) 12 D) 13 E) 14

**Q3:**

| | | |
|---|---|---|
| $(-3)^2$ | A | $-(-2)^3$ |
| $-5^2$ | $1^{2020}$ | $2^4$ |
| $(-2)^3$ | $(-19)^0$ | $(-1)^{191}$ |

**If the total sum of the numbers in all the boxes is 5, what is the value of A?** **(1 point)**

A) –4 B) 0 C) 4 D) 8 E) 10

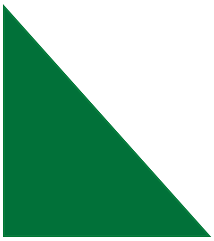
**Q4:**

$$\frac{0.6}{\sqrt{0.9}} + \frac{1.6}{\sqrt{0.4}}$$

**What is the result?** **(1 point)**

A) $\sqrt{10}$ B) $\frac{\sqrt{10}}{2}$ C) 1 D) $\frac{\sqrt{10}}{4}$ E) $\frac{1}{\sqrt{10}}$

**Q5:**

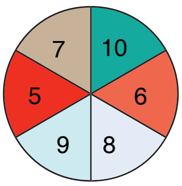


In a right triangle, the legs are in the ratio 5:12.

**If the perimeter of the triangle is 60 units, what is the length of the hypotenuse? (1 point)**

A) 6 B) 13 C) 20 D) 26 E) 39

**Q6:**




The target board shown has segments labeled with the points 5, 6, 7, 8, 9, and 10. A total of 5 shots will be fired, and for each shot, the score of the segment hit is added to the total score.

**How many different total scores can be achieved, knowing that all shots hit one of the segments? (2 points)**

A) 26 B) 25 C) 24 D) 21 E) 20

**Q7:**

A worker works 5 weekdays (Monday to Friday) for 9 hours per day and on Saturday for 4 hours. This worker can complete 1 piece of work in 4 hours.

**If the worker starts working on Monday, on what day will they finish 170 pieces of work? (2 points)**

A) Monday B) Tuesday
C) Wednesday D) Thursday
E) Friday

**Q8:** Bacteria multiply by splitting into two.

**If 1 bacterium fills a jar in 6 days, how many days will it take for 2 bacteria to fill the same jar? (2 points)**

A) 3 B) 4 C) 5 D) 6 E) 7

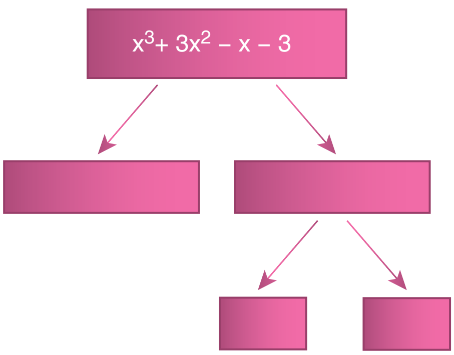
**Q9:**

| x | y |
|---|---|
| 1 | 1 |
| 2 | 3 |
| 0 | 1 |
| –2 | 7 |

**Based on the table of values above, which of the following is the rule for the function $y = f(x)$? (2 points)**

A) $y = 3x - 2$
B) $y = x^2 + 2$
C) $y = x^2 - x + 1$
D) $y = 2x - 1$
E) $y = 3x + 1$

**Q10:**




In the diagram above, the factors of the expression $x^3 + 3x^2 - x - 3$ are written in the corresponding boxes below.

**Which of the following expressions is not one of the factors written in the boxes? (2 points)**

A) $x + 3$
B) $x - 1$
C) $x + 1$
D) $x^2 - 1$
E) $x - 3$

**Q11:**

A certain number of walnuts are distributed equally among 40 students. If the boys receive 10 more walnuts each, and the girls receive 20 fewer walnuts each, then 50 walnuts remain undistributed.

**How many of the students are girls? (3 points)**

A) 25 B) 18 C) 17 D) 16 E) 15

**Q12:**

$$\frac{5x + 4}{x^2 + x - 2} = \frac{A}{x + 2} + \frac{B}{x - 1}$$

**What is the product of A.B? (3 points)**

A) 2 B) 3 C) 5 D) 6 E) 12

**Q13:**

In a table tennis tournament, each participant can play exactly one match with another participant. The number of matches played among the girls is 28. The number of matches played among the boys is 55.

**How many matches can be played between 1 boy and 1 girl? (3 points)**

A) 108 B) 96 C) 88 D) 112 E) 72

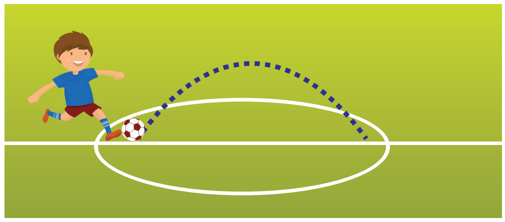
**Q14:**


To make coffee, 40 grams of coffee, 30 grams of sugar, and 190 grams of water are mixed together. When boiled, 10 grams of water evaporates.

**After cooking, what is the percentage of sugar in the mixture? (3 points)**

A) 6 B) 8 C) 10 D) 12 E) 14

**Q15:**



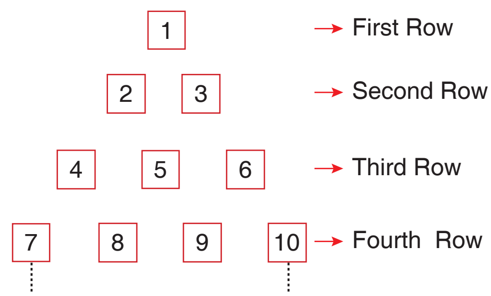
Alex kicks the ball, and the ball follows a parabolic trajectory as shown in the diagram. The ball's height f(x), in meters, after x seconds is given by the function:

$f(x) = -x^2 + 8x$

**How many seconds after Alex kicks the ball will its height be 12 meters? (3 points)**

A) 3 B) 4 C) 5 D) 6 E) 7

**Q16:**




**In the number pyramid shown above, what is the largest element in the 15th row (4 points)**

A) 124 B) 120 C) 108 D)102 E) 96

**Q17:**

$$a_n = \begin{cases} 5^{n+1} & \text{, if n is even,} \\ 4^{1-n} & \text{, if n is odd,} \end{cases}$$

**Find the common ratio of the sequence $(a_{2n-1})$. (4 points)**

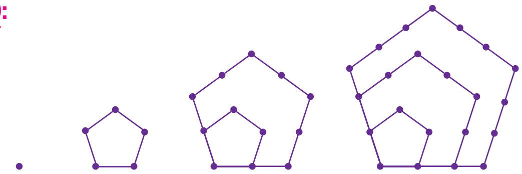
A) 16 B) 4 C) 2 D) $\frac{1}{4}$ E) $\frac{1}{16}$

**Q18:** $x < \log_{10}123456789 < x + 1$

**Find the value of $2x + 5$. (4 points)**

A) 22 B) 21 C) 20 D) 19 E) 18

**Q19:**



The first four terms of a pentagonal number sequence are shown above. The number of purple dots in the pattern corresponds to the terms of the sequence $a_n$.

**Based on this, what is the 5th term of the sequence $a_n$? (4 points)**

A) 22 B) 28 C) 35 D) 40 E) 42

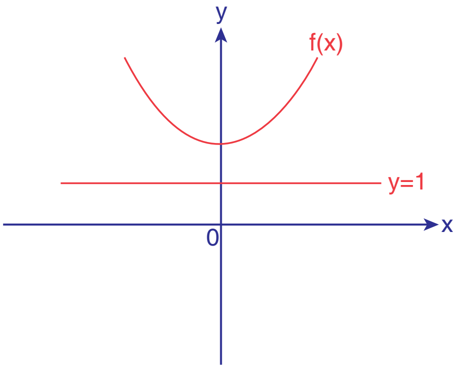
**Q20:**

The cube-shaped die shown above has 12 edges. Three of these edges are randomly painted blue.

**What is the probability that all three blue edges belong to the same face of the die? (4 points)**

A) $\frac{7}{45}$ B) $\frac{6}{55}$ C) $\frac{9}{55}$ D) $\frac{8}{45}$ E) $\frac{3}{110}$

**Q21:**




The function $f(x) = x^2 + 2(a - 1)x + 2a + 2$ represents a parabola, and its graph is given along with the line $y = 1$.

**Determine the range of a such that the parabola does not intersect the line y = 1. (5 points)**

A) $(-\infty, 4)$
B) $(0, \infty)$
C) $[0, 4]$
D) $[4, \infty)$
E) $(0, 4)$

**Q22:** $\dfrac{1 - \cos(2x)}{1 + \cos(2x)}$

**Simplify the expression. (5 points)**

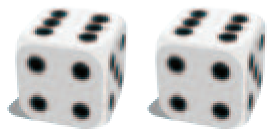
A) $\cot^2 x$
B) $\tan^2 x$
C) $\sin x$
D) $\cos x$
E) 1

**Q23:** The circumference of a circular sector with a radius r is 80 units.

**Which of the following functions represents the area of this circular sector? (5 points)**

A) $r.(80 - 2r)$
B) $(40 - r)^2$
C) $r.(40 - r)$
D) $r^2.40$
E) $r.(80 - r)$

**Q24:**



Two dice are rolled, and the sum of the numbers on their top faces is known to be 8.

**What is the probability that both numbers are prime? (5 points)**

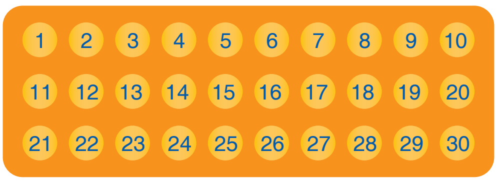
A) $\frac{3}{4}$
B) $\frac{2}{3}$
C) $\frac{1}{5}$
D) $\frac{2}{5}$
E) $\frac{1}{6}$

**Q25:** In the coordinate plane, the points A(3, 1), B(2, 6), and C(a,4) are given.

**If these points do not form a triangle, what is the value of a? (5 points)**

A) $\frac{11}{5}$ B) $\frac{12}{5}$ C) $\frac{13}{5}$ D) $\frac{14}{5}$ E) $\frac{16}{5}$

**Q26:**




In a lighting control panel, there are 30 buttons numbered from 1 to 30. When a button is pressed:

- If it is off, it turns on.
- If it is on, it turns off.

Initially, all buttons are off. The following process is applied:

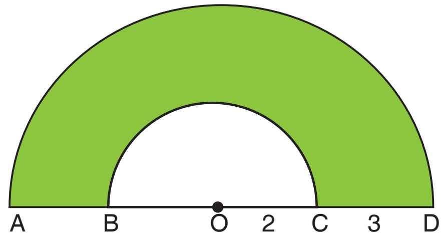
1. All buttons are pressed once (all turn on).
2. Then, every button that is a multiple of 2 is pressed (toggle state).
3. Next, every button that is a multiple of 3 is pressed, and so on.

This continues until the buttons that are multiples of 30 are pressed.

**How many buttons remain on at the end? (6 points)**

A) 8 B) 7 C) 6 D) 5 E) 4

**Q27:**




The point O is the center of two circles with diameters [AD] and [BC]:

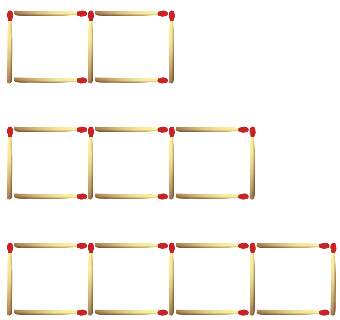
- |OC| = 2 cm
- |OD| = 5 cm

When the shaded green area is revolved 360° around the diameter [AD], it forms a solid.

**What is the volume of the resulting solid? (6 points)**

A) 408 $cm^3$ B) 448 $cm^3$
C) 458 $cm^3$ D) 468 $cm^3$
E) 478 $cm^3$

**Q28:**



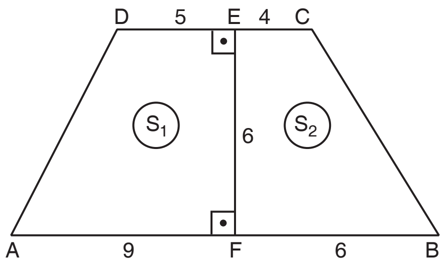
Using matchsticks of equal length:

- A 1 × 2 rectangular grid requires 7 matchsticks,
- A 1 × 3 rectangular grid requires 10 matchsticks,
- A 1 × 4 rectangular grid requires 13 matchsticks.

**How many matchsticks are needed to construct a 1 × 29 rectangular grid? (6 points)**

A) 70 B) 78 C) 88 D) 100 E) 120

**Q29:**




In the trapezoid ABCD:

|DE| = 5 cm

|EC| = 4 cm

|EF| = 6 cm

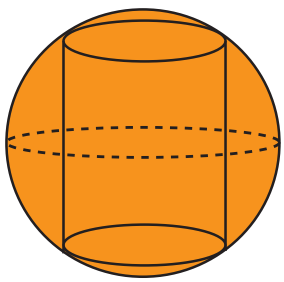
|AF| = 9 cm

|FB| = 6 cm

**Find the value of $S_1 - S_2$. (6 points)**

A) 8 B) 9 C) 10 D) 12 E) 15

**Q30:**



A cylinder with the largest possible volume is placed inside a sphere with a radius of 5 cm. The cylinder has a base radius of 3 cm.

**What is the surface area of the cylinder? (6 points)**

A) $48\pi$ cm² B) $54\pi$ cm²

C) $57\pi$ cm² D) $60\pi$ cm²

E) $66\pi$ cm²

**Q31:** The equation of a circle is given as:

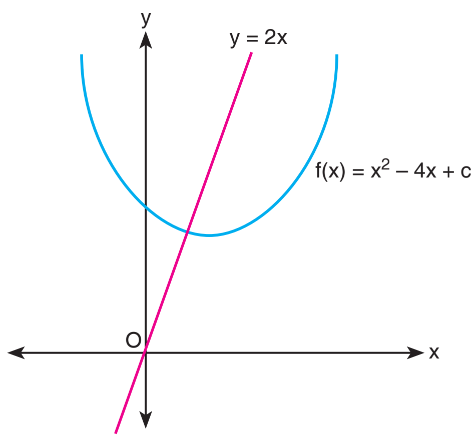
$$(x - 2)^2 + (y + 4)^2 = 8$$

**Find the length of the tangent drawn from the point A(6, 1) to the circle. (7 points)**

A) $\sqrt{29}$ B) $\sqrt{30}$ C) $4\sqrt{2}$

D) $\sqrt{33}$ E) $\sqrt{34}$

**Q32:**

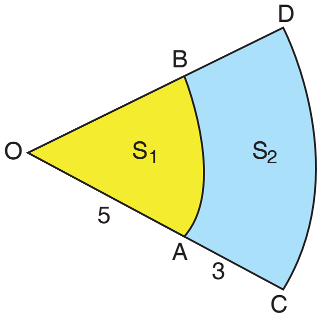



The line $y = 2x$ passes through the vertex of the parabola $f(x) = x^2 - 4x + c$.

**Find the value of c. (6 points)**

A) 4 B) 6 C) 8 D) 10 E) 12

**Q33:**



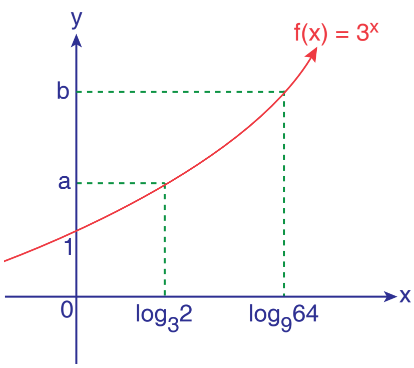

In the figure, O is the center of the circle, and circular sectors are shown:

- $S_1$ is the area of the yellow sector.
- $S_2$ is the area of the blue sector.

**What is the ratio $\frac{S_1}{S_2}$? (7 points)**

A) 1   B) $\frac{5}{8}$   C) $\frac{25}{39}$   D) $\frac{25}{64}$   E) $\frac{39}{64}$

**Q34:**

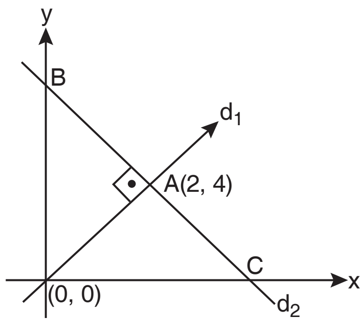



The graph of the function $f(x) = 3^x$ is shown. Two points on the graph are labeled with their abscissas and ordinates:

**What is the ratio $\frac{b}{a}$? (7 points)**

A) 1   B) 2   C) 4   D) 8   E) 16

**Q35:**




In the coordinate plane, A(2, 4), the lines $d_1$ and $d_2$ are perpendicular ($d_1 \perp d_2$).

**Find the equation of line $d_2$. (7 points)**

A) $2y + x - 10 = 0$
B) $2x + y - 8 = 0$
C) $3x + y - 12 = 0$
D) $2x + y - 10 = 0$
E) $3y + x - 12 = 0$

# GRADE 11-12 QUESTIONS AND SOLUTİONS

**ANSWER IS C**

**SOLUTION:**

**Q1:** The LCM between two numbers is their least common multiple, so K, L, and M must be factors of 120. The prime factorization of 120 is:

$120 = 2^3 \times 3 \times 5$

The factors of 120 are:

$\{1,2,3,4,5,6,8,10,12,15,20,24,30,40,60,120\}$

To maximize K + L + M, we need to choose the largest distinct factors of 120 that satisfy the LCM condition. The condition states that the LCM of each pair (K, L), (L, M), and (M, K) must be 120.

The maximum possible value of K + L + M is achieved with the distinct values:

$K = 40, L = 60, M = 120$

Which satisfy the conditions:

$LCM(40, 60) = 120,\ LCM(60,120) = 120,$

$LCM(120, 40) = 120$

The total sum is:

$K + L + M = 40 + 60 + 120 = 220$

**ANSWER IS E**

**SOLUTION:**

**Q2: Left Path:**

Left Path Result = $(x - 3) \times 6 = 6x - 18$

**Right Path:**

Right Path Result = $6 + (x + 7) = x + 13$

Combine Both Results:

Total Result = $(6x - 18) + (x + 13) = 93$

Simplify the equation:

$6x - 18 + x + 13 = 93$

$7x - 5 = 93$

$7x = 98$

$x = 14$

**ANSWER IS C**

**SOLUTION:**

**Q3: First Row:**

- $(-3)^2 = 9$
- A = A (unknown)
- $-(-2)^3 = 8$

**Second Row:**

- $2^4 = 16$
- $1^{2020} = 1$
- $-5^2 = -25$

**Third Row:**

- $(-2)^3 = -8$
- $(-19)^0 = 1$
- $(-1)^{191} = -1$

Total Sum = (First Row) + (Second Row) + (Third Row)

Total Sum = $9 + A + 8 + 16 + 1 + -25 + -8 + 1 + -1$

$\Rightarrow 1 + A = 5$

$A = 4$

The answer is C)4.

**ANSWER IS A**

**SOLUTION:**

**Q4:** We simplify each term individually.

**First Term:** $\frac{0.6}{\sqrt{0.9}}$

Rewrite 0.9 as $\frac{9}{10}$, so:

$$\sqrt{0.9} = \sqrt{\frac{9}{10}} = \frac{3}{\sqrt{10}}$$

Thus:

$$\frac{0.6}{\sqrt{0.9}} = \frac{0,6}{\frac{3}{\sqrt{10}}} = 0,6.\frac{\sqrt{10}}{3} = \frac{0,6\sqrt{10}}{3}$$

Simplify 0.6 ÷ 3:

$$\frac{0,6\sqrt{10}}{3} = \frac{\sqrt{10}}{5}$$

**Second Term:** $\frac{1,6}{\sqrt{0.4}}$

Rewrite 0.4 as $\frac{4}{10}$, so:

$$\sqrt{0.4} = \frac{\sqrt{4}}{\sqrt{10}} = \frac{2}{\sqrt{10}}$$

Thus:

$$\frac{1,6}{\sqrt{0.4}} = \frac{1.6}{\frac{2}{\sqrt{10}}} = 1.6.\frac{\sqrt{10}}{2} = \frac{1.6}{\frac{2}{\sqrt{10}}}$$

Simplify 1.6 ÷ 2:

$$\frac{1.6\sqrt{10}}{2} = 0.8\sqrt{10}$$

Now combine the results:

$$\frac{\sqrt{10}}{5} + 0.8\sqrt{10}$$

Rewrite 0.8 as $\frac{4}{5}$, so:

$$\frac{\sqrt{10}}{5} + \frac{4\sqrt{10}}{5} = \frac{1\sqrt{10} + 4\sqrt{10}}{5} = \frac{5\sqrt{10}}{5}$$

Simplify:

$$\frac{5\sqrt{10}}{5} = \sqrt{10}$$

**ANSWER IS D**

**SOLUTION:**

**Q5:** Since the legs are in the ratio 5 : 12, let the sides of the triangle be:

- 5k (first leg),
- 12k (second leg),
- 13k (hypotenuse, based on the Pythagorean triplet 5 : 12: 13)

The perimeter of the triangle is given as 60.

The sum of the three sides is:

$$5k + 12k + 13k = 60$$

Simplify: 30k = 60

Solve for k: = 2

The hypotenuse is 13k

Substituting k = 2: Hypotenuse = 13 x 2 = 26

**ANSWER IS A**

**SOLUTION:**

**Q6:** Each shot hits a segment, and we can repeat scores (e.g., hitting "10" multiple times).

If $x_1, x_2, x_3, x_4, x_5$ are the scores of the 5 shots, the total score is:

$$S = x_1 + x_2 + x_3 + x_4 + x_5$$

Where $x_i \in \{5, 6, 7, 8, 9, 10\}$ for all i.

The minimum total score is achieved when all 5 shots hit "5":

$$S_{min} = 5 + 5 + 5 + 5 + 5 = 25.$$

The maximum total score is achieved when all 5 shots hit "10":

$$S_{max} = 10 + 10 + 10 + 10 + 10 = 50$$

The total score depends on the number of ways the shots can add up, with repetition allowed. For any score between 25 and 50, we check whether it can be achieved using a combination of the values {5, 6, 7, 8, 9, 10}.

There are 26 unique total scores that can be achieved by firing 5 shots on the target board.

The possible scores range from 25 (minimum) to 50 (maximum).

**ANSWER IS E**

**SOLUTION:**

**Q7:** Pieces of work completed:

- Weekdays (Monday to Friday):

$$\frac{9\,\text{hours}}{\text{day}} \div \frac{4\,\text{hours}}{\text{piece}} = \frac{2.25\,\text{pieces}}{\text{day}}$$

- Saturday: $\frac{4\,\text{hours}}{\text{day}} \div \frac{4\,\text{hours}}{\text{piece}} = \frac{1\,\text{pieces}}{\text{day}}$

The worker completes:

- 5 weekdays: $2.25 \times 5 = 11.25$ pieces
- Saturday: 1 piece
- Total weekly work capacity:

$$11.25 + 1 = \frac{12.25\,\text{pieces}}{\text{week}}$$

To complete 170 pieces, calculate the number of full weeks needed:

$$\text{Full Weeks} = \frac{170}{12.25} = 13\,\text{weeks}$$

In 13 weeks, the worker completes:

$$13 \times 12.25 = 159.25\,\text{pieces}.$$

Remaining work after 13 weeks:

$$170 - 159.25 = 10.75\,\text{pieces}$$

After 13 weeks, the worker begins the 14th week on Monday. We calculate day – by– day:

Monday: Completes 2.25 pieces

Remaining: 10.75 – 2.25 = 8.5 pieces

Tuesday: Completes 2.25 pieces

Remaining: 8.5 – 2.25 = 6.25 pieces

Wednesday: Completes 2.25 pieces

Remaining: 6.25 – 2.25 = 4.0 pieces

Thursday: Completes 2.25 pieces

Remaining: 4.0 – 2.25 = 1.75 pieces

Friday: Completes 1.75 pieces

Remaining: 0 pieces

The worker finishes all 170 pieces on Friday of the 14th week.

**ANSWER IS C**

**SOLUTION:**

**Q8:** Bacteria double their population each time they split. 1 bacterium takes 6 days to fill the jar. This means the jar reaches its full capacity when the final doubling occurs on the 6th day. If 2 bacteria are present at the start, the jar will reach its full capacity one doubling period earlier, because the starting population is already doubled. Each doubling period halves the time remaining to fill the jar.

Since it takes 6 days for 1 bacterium, starting with 2 bacteria will reduce the time by 1 doubling period: $6 - 1 = 5$ days.

**ANSWER IS C**

**SOLUTION:**

**Q9:** We test each given function to see if it satisfies the table.

**Option A: $y = 3x - 2$**

Substitute the x-values:

- For $x = 1$: $y = 3(1) - 2 = 1$ (Matches)
- For $x = 2$: $y = 3(2) - 2 = 4$
  **(Does not match $y = 3$)**

This rule is **incorrect**

**Option B: $y = x^2 + 2$**

Substitute the x-values:

- For $x = 1$: $y = 1^2 + 2 = 3$ **(Does not match $y= 1$)**

This rule is incorrect

**Option C: $y = x^2 - x + 1$**

Substitute the x-values:

- For $x = 1$: $y = 1^2 - (1) + 1 = 1$ (Matches)
- For $x = 2$: $y = 2^2 - (2) + 1 = 3$ (Matches)
- For $x = 0$: $y = 0^2 - (0) + 1 = 1$ (Matches)
- For $x = -2$: $y = (-2)^2 - (-2) +1 = 4 + 2 + 1 = 7$ (Matches)

This rule is **correct**

**Option D: $y=2x-1$**

Substitute the x-values:

- For $x = 1$: $y = 2(1) - 1 = 1$ (Matches)
- For $x = 2$: $y = 2(2) - 1 = 3$ (Matches).
- For $x = 0$: $y = 2(0) -1 = -1$
  **(Does not match $y = 1$)**

This rule is **incorrect**

**Option E: $y = 3x + 1$**

Substitute the x-values:

- For $x = 1$: $y = 3(1) + 1 = 4$
  **(Does not match $y = 1$)**

This rule is **incorrect**

**ANSWER IS E**

**SOLUTION:**

**Q10:** The given polynomial is $x^3 + 3x^2 - x - 3$.

Group terms:

$$(x^3 + 3x^2) - (x + 3)$$

Factor each group:

$$x^2(x + 3) - 1(x + 3)$$

Factor out (x + 3):

$$(x + 3)(x^2 - 1)$$

The second term $x^2 - 1$ is a difference of squares:

$$x^2 - 1 = (x - 1)(x + 1)$$

Thus, the fully factored form of the polynomial is:

$$x^3 + 3x^2 - x - 3 = (x + 3)(x - 1)(x + 1)$$

The factors are:

- $x + 3$
- $x - 1$
- $x + 1$

From the given options:

- $x^2 - 1$ is valid because it combines $(x - 1)(x + 1)$.
- $x - 3$ is NOT a factor.

**ANSWER IS E**

**SOLUTION:**

**Q11:** DLet:

- x: Number of girls.
- y = 40 – x: Number of boys (since there are 40 students total).
- N: Total number of walnuts.

The walnuts are initially distributed equally among all 40 students:

Each student gets $\frac{N}{40}$ walnuts

Each boy receives 10 more walnuts:

Each boy gets $\frac{N}{40} + 10$

Each girl receives 20 fewer walnuts:

Each girl gets $\frac{N}{40} - 20$

After this redistribution, 50 walnuts are left over.

The total walnuts distributed are:

$$\text{Walnuts distributed} = y\left(\frac{N}{40} + 10\right) + x\left(\frac{N}{40} - 20\right)$$

The equation for the remaining walnuts is:

$$N - \left[y\left(\frac{N}{40} + 10\right) + x\left(\frac{N}{40} - 20\right)\right] = 50$$

Substitute y = 40 – x:

$$N - \left[\left(40 - x\right)\left(\frac{N}{40} + 10\right) + x\left(\frac{N}{40} - 20\right)\right] = 50$$

Expand:

$$N - \left[\left(40 - x\right).\frac{N}{40} + (40 - x).10 + x.\frac{N}{40} - x.20\right] = 50$$

Distribute terms:

$$N - \left[\left(40 - x\right)\frac{N}{40} + 400 - 10x + x\frac{N}{40} - 20x\right] = 50$$

Combine like terms:

$$N - \left[\left(\frac{40N - xN + xN}{40 + 400 - 30x}\right)\right] = 50$$

Simplify further:

$$N - [N + 400 - 30x] = 50$$

$$N - N - 400 + 30x = 50$$

$$30x = 450$$

$$x = 15$$

15 girls are in the group.

**ANSWER IS D**

**SOLUTION:**

**Q12: Step 1: Factorize the Denominator**

The denominator $x^2 + x - 2$ can be factorized:

$x^2 + x - 2 = (x + 2)(x - 1)$

Thus, the given equation becomes:

$$\frac{5x + 4}{(x + 2)(x - 2)} + \frac{A}{x + 2} + \frac{B}{x - 1}$$

**Step 2: Combine the Right-Hand Side**

The right-hand side can be expressed with a common denominator:

$$\frac{A}{x+2} + \frac{B}{x-1} = \frac{A(x - 1) + B(x + 2)}{(x + 2)(x - 1)}$$

Equating the numerators:

$5x + 4 = A(x - 1) + B(x + 2)$

**Step 3: Expand and Simplify**

Expand the terms on the right-hand side:

$A(x - 1) + B(x + 2) = Ax - A + Bx + 2B$

Combine like terms:

$(A + B)x + (-A + 2B)$

Thus:

$5x + 4 = (A + B)x + (-A + 2B)$

**Step 4: Solve for A and B**

Equate coefficients of x and the constant term:

Coefficient of x:

$A + B = 5$

Constant term:

$-A + 2B = 4$

**Step 5: Solve the System of Equations**

From $A + B = 5$, we can express A as: $A = 5 - B$

Substitute $A = 5 - B$ into $- A + 2B$
$= 4{:}-(5 - B) + 2B = 4$

Simplify:

$-5 + B + 2B = 4$

$3B = 9 \Rightarrow B = 3$

Substitute $B = 3$ into $A + B = 5$:

$A + 3 = 5 \Rightarrow A = 2$

$A.B = 2.3 = 6$

**ANSWER IS C**

**SOLUTION:**

**Q13:** The number of matches played among n participants is given by:

$$\text{Number of Matches} = \binom{n}{2} = \frac{n(n-1)}{2}$$

For Girls: The number of matches among the girls is 28. Solve:

$$\frac{n(n-1)}{2} = 28$$

Multiply through by 2:

$n(n-1) = 56$

Solve the quadratic equation:

$n^2 - n - 56 = 0$

Factoring:

$(n - 8)(n + 7) = 0$

Since $n > 0$:

$n = 8$

There are 8 girls.

For Boys: The number of matches among the boys is 55. Solve:

$$\frac{n(n-1)}{2} = 55$$

Multiply through by 2:

$n(n - 1) = 110$

Solve the quadratic equation:

$n^2 - n - 110 = 0$

Factoring:

$(n - 11)(n + 10) = 0$

Since $n > 0$:

$n = 11$

There are 11 boys.

Each boy can play with every girl. The total number of matches is:

Total Matches = (Number of Boys) × (Number of Girls)

Substitute the values:

Total Matches = $11 \times 8 = 88$

**ANSWER IS D**

**SOLUTION:**

**Q14: Step 1: Total Initial Weight**

The initial total weight of the mixture is:
40 (coffee) + 30 (sugar) + 190 (water) = 260 grams

**Step 2: Total Weight After Evaporation**

When 10 grams of water evaporates, the remaining weight of the mixture becomes:

$260 - 10 = 250$ grams

**Step 3: Sugar Percentage in the Mixture**

The sugar content remains 30 grams, as evaporation does not affect the sugar. The percentage of sugar in the final mixture is:

$$\text{Sugar Percentage} = \left(\frac{\text{Sugar Weight}}{\text{Total Weight After Evaporation}}\right) \times 100$$

Substitute the values:

$$\text{Sugar Percentage} = \left(\frac{30}{250}\right) \times 100$$

Simplify:

Sugar Percentage = 12%.

The answer is D.

**ANSWER IS D**

**SOLUTION:**

**Q15:** The height function is given as:

$f(x) = -x^2 + 8x$

Set $f(x) = 12$ to find the time x:

$-x^2 + 8x = 12$

Rearrange the equation:

$-x^2 + 8x - 12 = 0$

Multiply through by $-1$ to simplify:

$x^2 - 8x + 12 = 0$

Factorize:

$x^2 - 8x + 12 = (x - 6)(x - 2) = 0$

Thus:

$x = 6$ or $x = 2$

The ball reaches 12 meters twice:

At $x = 2$: While rising.

At $x = 6$: While falling.

The question asks for the maximum time: $x = 6$ seconds

**ANSWER IS B**

**SOLUTION:**

**Q16:** From the given pyramid:

1. **st row:** 1
2. **nd row:** 2, 3
3. **rd row:** 4, 5, 6
4. **th row:** 7, 8, 9, 10

Each new row adds consecutive numbers, starting where the previous row left off.

**Start of a row ($S_n$):**

- The starting number of the n-th row is the sum of the first n − 1 rows: $S_n = \frac{n(n+1)}{2} + 1$

**End of a row ($E_n$):**

- The ending number of the nnn-th row is:

$$E_n = \frac{n(n+1)}{2}$$

The largest element in the 15th row is the end of the 15th row ($E_{15}$).

Using the formula for $E_n$:

$$E_n = \frac{n(n+1)}{2}$$

Substitute n = 15: $E_{15} = \frac{15 \times 16}{2} = \frac{240}{2} = 120$

**ANSWER IS E**

**SOLUTION:**

**Q17:** For n = 1, 2, 3, … 2n − 1 sequence is always an odd number.

- When n is odd in $a_n$, the formula $a_n = 4^{1-n}$ applies.
- Therefore: $a_{2n-1} = 4^{1-(2n-1)} = 4^{-2n+2} \Rightarrow a_{(2n-1)} = 4^{2-2n}$

The general term of $a_{2n-1}$ is $4^{2-2n}$.

To find the common ratio, divide consecutive terms:

$$r = \frac{a_{2n}}{a_{2n-1}} = \frac{4^{2-2(n+1)}}{4^{2-2n}}$$

Simplify the exponent: $r = 4^{2-2(n+1)-(2-2n)} = 4^{-2}$

Therefore: $r = \frac{1}{16}$

The common ratio of the sequence ($a_{2n-1}$) is $\frac{1}{16}$.

**ANSWER IS B**

**SOLUTION:**

**Q18: Approximate** $\log_{10} 123456789$:

123456789 is approximately $1.23456789 \times 10^8$.

Using logarithmic properties:

- $\log_{10} 123456789 = \log_{10}(1.23456789 \times 10^8)$
  $= \log_{10} 1.23456789 + \log_{10} 10^8$
- $\log_{10} 10^8 = 8$
- $\log_{10} 1.23456789$ is slightly greater than 0.

Therefore:

$\log_{10} 123456789 \approx 8$

From the inequality $x < \log_{10} 123456789 < x + 1$, and since $\log_{10} 123456789 \approx 8$, we conclude: x = 8.

Substitute x = 8: 2x + 5 = 2(8) + 5 = 16 + 5 = 21

The answer is B.

**ANSWER IS C**

**SOLUTION:**

**Q19:** The sequence $a_n$ is 1, 5, 12, 22,…

Let's examine the differences:

1. **First Differences:**
   5 − 1 = 4,
   12 − 5 = 7,
   22 − 12 = 10.
2. **Second Differences:**
   7 − 4 = 3,
   10 − 7 = 3.

Since the second differences are constant, the sequence follows a quadratic pattern.

Formula for Pentagonal Numbers:

The general formula for the n-th pentagonal number is:

$$a_n = \frac{n(3n-1)}{2}$$

Substitute n = 5 into the formula:

$$a_5 = \frac{5(3.5-1)}{2} = \frac{5(15-1)}{2} = 5.\frac{14}{2} = \frac{70}{2} = 35$$

**ANSWER IS B**

**SOLUTION:**

**Q20:** The cube has 12 edges.

To randomly select 3 edges from the total of 12:

Total combinations = $\binom{n}{2} = \frac{12.11.10}{3.2.1}$

$\frac{12.11.10}{3.2.1} = 220$

Each face of the cube has 4 edges, and we need to select 3 edges from one face.

Number of ways to choose 3 edges from 4 edges on one face: $= \frac{4}{1} = 4$

Since there are 6 faces on the die, the total number of favorable outcomes is: 6.4 = 24.

Probability $= \frac{\text{Favorable outcomes}}{\text{Total outcomes}}$ : $P = \frac{24}{220} = \frac{6}{55}$

**ANSWER IS E**

**SOLUTION:**

**Q21:** We need $f(x) = x^2 + 2(a - 1)x + 2a + 2$ to not intersect the line $y = 1$.

The quadratic equation $f(x) = 1$ must have no real solutions. This happens when the discriminant ($\Delta$) is negative.

**Set f(x) =1:**

Simplify:

$x^2 + 2(a - 1)x + 2a + 2 = 0.$

The discriminant for a quadratic equation is:

$\Delta = b^2 - 4ac$

Here:

- $b = 2(a - 1)$
- $a = 1$
- $c = 2a + 1$

Substitute into $\Delta$:

$\Delta = [2(a - 1)]^2 - 4(1)(2a + 1)$

Expand:

$\Delta = [4(a- - 1)^2 - 8a - 4$

Factorize:

$\Delta = 4a(a - 4)$

**Solve $\Delta < 0$:**

For $\Delta < 0$:

$4a(a - 4) < 0$

Divide by 4:

$a(a - 4) < 0$

Solve the inequality:

Critical points are $a = 0$ and $a = 4$

The inequality $a(a - 4) < 0$ holds when $0 < a < 4$

The answer is E.

**ANSWER IS B**

**SOLUTION:**

**Q22: Use Trigonometric Identities for cos.(2x):**

$\cos(2x) = 1 - 2\sin^2(x)$

Substitute $\cos(2x) = 1 - 2\sin^2(x)$ into the expression:

$$\frac{1-\cos(2x)}{1+\cos(2x)} = \frac{1-(1-2.\sin^2(x)}{1+(1-2\sin^2(x)}$$

Simplify Numerator and Denominator:

Numerator:

$$1-\left(1-2\sin^2(x)\right) = 2\sin^2(x)$$

Denominator:

$$1+\left(1-2\sin^2(x)\right) = 2-2\sin^2(x) = 2\cos^2(x)$$

The expression becomes:

$$\frac{2\sin^2(x)}{\cos^2(x)}$$

Cancel the 2's:

$$\frac{\sin^2(x)}{\cos^2(x)} = \tan^2 x$$

**ANSWER IS C**

**SOLUTION:**

**Q23:** The circumference of a circular sector is given as:

Circumference = 80

For a circular sector, the circumference includes:

- Two radii (2r),
- The arc length of the sector (s).

Thus:

$2r + s = 80$

From this, the arc length s is:

$s = 80 - 2r$

The area A of a circular sector is given by:

$$A = \frac{1}{2}r.s$$

Where r is the radius and s is the arc length.

Substitute $s = 80 - 2r$ into the formula:

$$A = \frac{1}{2}r.(80-2r)$$

Simplify:

$A = r.(40 - r)$

The answer is C.

**ANSWER IS D**

**SOLUTION:**

**Q24:** The numbers on a die are 1, 2, 3, 4, 5, 6. Among these, the prime numbers are 2,3,5.

We need pairs of dice rolls that sum to 8. These pairs are (2, 6), (3, 5), (4, 4), (5, 3), (6, 2).

Thus, there are 5 possible pairs.

From the list of pairs:

- (2, 6): 6 is not prime.
- (3, 5): Both are prime.
- (4, 4): 4 is not prime.
- (5, 3): Both are prime.
- (6, 2): 6 is not prime.

Only the pairs (3, 5) and (5, 3) satisfy the condition that both numbers are prime.

Thus, there are 2 favorable outcomes.

The probability is the ratio of favorable outcomes to total outcomes:

$$P = \frac{\text{Number of Favorable Outcomes}}{\text{Total Number of Possible Pairs}} = \frac{2}{5}$$

**ANSWER IS B**

**SOLUTION:**

**Q25:** Three points do not form a triangle if they are collinear. For three points $A(x_1, y_1)$, $B(x_2, y_2)$, and $C(x_3, y_3)$ to be collinear, the area of the triangle they form must be zero.

The area of a triangle with vertices $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ is given by:

$$\text{Area} = \frac{1}{2}|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)|$$

Set the area to 0 for collinearity:

$$x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2) = 0$$

Substitute A(3, 1), B(2, 6), and C(a, 4) into the equation:

$$3(6-4) + 2(4-1) + a(1-6) = 0$$

Simplify:

$$3(2) + 2(3) + a(-5) = 0$$

$$6 + 6 - 5a = 0$$

$$12 - 5a = 0 \Rightarrow 5a = 12 \Rightarrow a = \frac{12}{5}$$

**ANSWER IS D**

**SOLUTION:**

**Q26:** A button's final state depends on how many times it is pressed. A button is pressed once for each of its divisors. For example:

- Button 12 is pressed for 1, 2, 3, 4, 6, 12 (6 divisors),
- Button 25 is pressed for 1, 5, 25 (3 divisors).

If a button is pressed an odd number of times, it will remain on. Otherwise, it will remain off.

A number has an odd number of divisors only if it is a perfect square. This is because divisors normally come in pairs (e.g., for 12: 1 × 12, 2 × 6, 3 × 4), but for perfect squares (e.g., 16: 1 × 16, 2 × 8, 4 × 4), one divisor is repeated.

The perfect squares within the range are:

$1^2 = 1$
$2^2 = 4$
$3^2 = 9$
$4^2 = 16$
$5^2 = 25$

Thus, there are 5 buttons that will remain on: 1, 4, 9, 16, 25

**ANSWER IS D**

**SOLUTION:**

**Q27:** The formula for the volume of a sphere is:

$$V = \frac{4}{3}\pi r^3$$

Where r is the radius of the sphere.

The radius of the outer sphere is $r_{outer} = 5\,cm$. Substituting into the formula:

$$V_{outer} = = \frac{4}{3}\pi r^3$$

Simplify:

$$V_{outer} = \frac{4}{3}\,3.125 = 500\ cm^3$$

The radius of the inner sphere is $r_{inner} = 2\,cm$. Substituting into the formula:

$$V_{inner} = \frac{4}{3}\pi r^3$$

Simplify:

$$V_{inner} = \frac{4}{3}\pi 8 = 32\ cm^3$$

The volume of the shaded region is the difference between the volumes of the outer and inner spheres:

$$V_{shaded} = V_{outer} - V_{inner}$$

Substitute the values:

$$V_{shaded} = 500\ cm^3 - 32\ cm^3 = 468\ cm^3$$

**ANSWER IS C**

**SOLUTION:**

**Q28:** The number of matchsticks required follows a pattern based on the number of cells n in the $1 \times n$ grid:

- For n = 2: 7 matchsticks,
- For n = 3: 10 matchsticks,
- For n = 4: 13 matchsticks.

The difference between consecutive terms is constant:

$10 - 7 = 3$
$13 - 10 = 3$

Thus, the number of matchsticks S(n) follows a linear equation:

$S(n) = 3n + c$

Using the given data (n = 2, S(2) = 7):

$7 = 3(2) + c \Rightarrow c = 7 - 6 = 1$

Thus, the equation for the number of matchsticks is:
$S(n) = 3n + 1$

Substitute n = 29 into the formula:

$S(29) = 3(29) + 1 = 87 + 1 = 88$

The answer is C)88.

**ANSWER IS D**

**SOLUTION:**

**Q29:** We need to find $S_1 - S_2$, where:

- $S_1$: The area of trapezoid ADEF
- $S_2$: The area of trapezoid FECB

The area of the trapezoid($S_1$) is:

$$\text{Area} = \frac{1}{2}.(\text{Base1} + \text{Base2}).\text{Height}$$

- **Base 1:** $|AF| = 9\,cm$
- **Base 2:** $|DE| = 5\,cm$
- **Height:** $|EF| = 6\,cm$

Substitute into the formula:

$$S_1 = \frac{1}{2}.(|AF| + |DE|).|EF|.$$

$$S_1 = \frac{1}{2}.(9 + 5).6.$$

$$S_1 = \frac{1}{2}.14.6 = 42\,cm^2$$

The area of the trapezoid($S_2$) is

**Base 1:** $|FB| = 6\,cm$

**Base 2:** $|EC| = 4\,cm$

**Height:** $|EF| = 6\,cm$

Substitute into the formula:

$$S_2 = \frac{1}{2}.(|FB| + |EC|).|EF|.$$

$$S_2 = \frac{1}{2}.(6 + 4).6.$$

$$S_2 = \frac{1}{2}.10.6 = 30\,cm^2$$

The difference between the areas is:

$S_1 - S_2 = 42\,cm^2 - 30\,cm^2 = 12\,cm^2.$

**ANSWER IS E**

**SOLUTION:**

**Q30:** The cylinder is inscribed within the sphere. The height of the cylinder can be found using the geometry of the sphere.

The sphere's diameter is twice the radius:

Diameter of sphere = 2.5 = 10 cm.

Since the cylinder is inscribed in the sphere, the diagonal of the cylinder corresponds to the diameter of the sphere. Using the Pythagorean theorem, we relate the cylinder's height h and radius r = 3 cm:

$h^2 + (2r)^2 = (\text{diameter of sphere})^2$

Substitute the known values:

$h^2 + (2.3)^2 = 10^2$

Simplify:

$h^2 + 36 = 100 \Rightarrow h^2 = 64 \Rightarrow h = 8\,\text{cm}.$

The height of the cylinder is h = 8 cm.

The surface area of a cylinder is:

$A = 2\pi r^2 + 2\pi rh$

Where r = 3 cm and h = 8 cm.

$2\pi r^2 = 2\pi(3)^2 = 2\pi.9 = 18\pi\ \text{cm}^2$

$2\pi rh = 2\pi(3)(8) = 48\pi\,\text{cm}^2.$

$A = 18\pi\ \text{cm}^2 + 48\pi\ \text{cm}^2 = 66\pi\,\text{cm}^2$

**ANSWER IS D**

**SOLUTION:**

**Q31:** The formula for the length of a tangent from a point $P(x_1, y_1)$ to a circle $(x - h)^2 + (y - k)^2 = r^2$ is:

Tangent length = $\sqrt{(x_1 - h)^2 + (y_1 - k)^2 - r^2)}$

**Identify the center and radius of the circle:**

- The center of the circle is (h, k ) = (2, −4)
- The radius squared is $r^2 = 8$

**Substitute the point A(6, 1):** Using $(x_1, y_1) = (6,1)$:

Tangent length = $\sqrt{(6-2)^2 + \left(1-(-4)\right)^2 - 8}$

$(6-2)^2 = 16$

$\left(1-(-4)\right)^2 = 5^2 = 25$

$r^2 = 8$

Substitute these values:

Tangent length = $\sqrt{(16 + 25 - 8)}$

Tangent length = $\sqrt{33}$

**ANSWER IS C**

**SOLUTION:**

**Q32:** The equation of the parabola is:

$$f(x) = x^2 - 4x + c$$

The x-coordinate of the vertex is found using the formula:

$$x_v = \frac{-b}{2a}$$

Here, a = 1 and b = −4:

$$x_v = -\frac{-4}{2} = \frac{4}{2} = 2$$

The x-coordinate of the vertex is x = 2.

Substitute x = 2 into the parabola equation:

$$f(2) = (2)^2 - 4(2) + c = 4 - 8 + c = -4 + c$$

The y-coordinate of the vertex is −4 + c.

The line y = 2x passes through the vertex. Substituting x = 2 into the line equation:

$$y = 2(2) = 4$$

Thus, the y-coordinate of the vertex is also 4.

Equating the y-coordinate from the parabola and the line:

$$-4 + c = 4.$$

$$c = 4 + 4 = 8$$

**ANSWER IS C**

**SOLUTION:**

**Q33:** We are tasked to find the ratio $\frac{S_1}{S_2}$, where:

- $S_1$ is the yellow area,
- $S_2$ is the blue area, which is the difference between the total area of the larger circle (radius 8) and the yellow area.

The radii given are:

$r_1 = 5$ for the yellow region

$r_2 = 8$ for the larger circle

Yellow Area $S_1$:

$$S_1 = r_1^2 = 5^2 = 25$$

Total Circle Area (radius $r_2 = 8$):

Total Area = $r_2^2 = 8^2 = 64$

Blue Area $S_2$: The blue area is the difference between the total circle area and the yellow area:

$$S_2 = 64 - 25 = 39$$

$$\frac{S_1}{S_2} = \frac{25}{39}$$

**ANSWER IS C**

**SOLUTION:**

**Q34:** The function $f(x) = 3^x$ is given. The points on the graph are:

- At $x = \log_3 2$, the ordinate is $a = 3^{\log_3 2} = 2$.
- At $x = \log_9 64$, the ordinate is b.

We need to find $\frac{b}{a}$.

We know:

$b = 3^{\log_9 64}$

Using the property $\log_9 64 = \frac{\log_3 64}{\log_3 9}$,

and since $\log_3 9 = 2$, we have:

$\log_9 64 = \frac{\log_3 64}{2}$

Substitute this back into the expression for b:

$b = 3^{\log_9 64} = 3^{\frac{\log_3 64}{2}}$

Using the property $a^{\frac{m}{n}} = \sqrt[n]{a^m}$, this simplifies to:

$b = \sqrt{3^{\log_9 64}}$

Since $3^{\log_9 64} = 64$, we find:

$b = \sqrt{64} = 8$

We already know $a = 2$ and $b = 8$.

Thus:

$\frac{b}{a} = \frac{8}{2} = 4$

**ANSWER IS A**

**SOLUTION:**

**Q35:** The line $d_1$ passes through (0,0) and A(2, 4). The slope of $d_1$ is:

$m_1 = \frac{4-0}{2-0} = 2$

Since $d_1.d_2$, the slopes of $d_1$ and $d_2$ satisfy:

$m_1.m_2 = -1$

Substituting $m_1 = 2$:

$2.m_2 = -1.m_2 = \frac{-1}{2}$

The slope of $d_2$ is $m_2 = \frac{-1}{2}$, and it passes through A(2,4).

The point-slope form of a line is: $y - y_1 = m(x - x_1)$.

Substituting $(x_1, y_1) = (2, 4)$ and $m = \frac{-1}{2}$:

$y - 4 = \frac{-1}{2}(x-2)$

Simplify:

$y - 4 = \frac{-1}{2}x + 1$

$y = \frac{-1}{2}x+5$

Multiply through by 2 to eliminate the fraction:

$2y = -x + 10$

Rearrange:

$x + 2y - 10 = 0$